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OPTIMIZATION OF OPTICAL DENSITY REQUIREMENTS FOR MULTI-WAVELENGTH LASER SAFETY

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INTRODUCTION

A multi wavelength laser can be defined as a laser system emitting more than one discrete wavelength simultaneously. If each wavelength is considered a separate “mode” of operation, then each of these modes may have differing beam diameters, divergence values or even time-intensity variations. In some instances these multi wavelength systems are the result of gas lasers operating in “all-line” configurations, or pulsed lasers equipped with non-linear optical devices which may produce fundamental, frequency-doubled and frequency-tripled output.

Safety parameters for multi wavelength lasers cannot be computed by using only a single analytical mathematical formula. The overall exposure is increased compared to that when only a single wavelength is emitted at a time, and relative exposure magnitudes may vary as a function of exposure distance or time. This prevents applying the common equations¹ for single-wavelength exposures to a multi wavelength laser problem. While previous work by Lyon² has described the basic methodology required to provide an approach to safety, it stops short of addressing all considerations. This paper extends the methods of Lyon² to determine optimal laser eye protection (LEP) design, which both minimizes overall attenuation and provides for optimization of user performance through the maximization of other metrics such as visible light transmittance.

Single-wavelength hazard analysis requires finding maximum permissible exposure (MPE) limits, accessible emission limits (AEL) and the effective amount of energy passing through the limiting aperture for each mode (Qf). This paper assumes that the reader is familiar with the techniques³ for determining these parameters according to the ANSI Z136.1-2000¹ Safety Standard and has a basic understanding of laser safety analysis methods and terminology. Similar constructs and methods are used as part of the process outlined here for multi wavelength analysis.

Here, we examine two approaches to the estimation of optical density (OD) requirements in a multi wavelength laser exposure. The first approach formulates the problem at a relatively high level using mathematical programming, formulated as a nonlinear optimization algorithm, and solved using GAMS⁴, a modeling language for representing the mathematical model. The second method accomplishes these goals through the development of a simplified algorithm, which we have implemented in the C++ programming language⁵ for the purposes of presenting comparative results.

PROBLEM DESCRIPTION

As described in Lyon², when one is exposed to a laser emitting multiple wavelengths simultaneously, the combined effect should be considered. In order to denote the various parameters required for the analyses, we will adopt a convention which is similar to the ANSI Z136.1 Standard. Let $\lambda \in SM$ (subset of modes) with the individual wavelengths emitted by the laser indexed by the symbol λ . The symbol $Q_{f\lambda}$ denotes the energy (in Joules) delivered through the corresponding limiting aperture at wavelength $\lambda \in SM$, and AEL_λ denotes the accessible emission limit (in Joules) for wavelength $\lambda \in SM$. This

accessible emission limit is computed from the product of the MPE, and the area of the appropriate limiting aperture. In absence of LEP, the safe exposure limit is considered to be exceeded if:

$$\sum_{\lambda \in SM} \frac{Qf_\lambda}{AEL_\lambda} > 1. \quad (1)$$

To avoid the situation described in Equation (1), LEP is used, and in determining optimal LEP we effectively select an optical density D_λ at each wavelength $\lambda \in SM$, which in turn yields a transmittance τ_λ via $\tau_\lambda = 10^{-D_\lambda}$, $\lambda \in SM$. To ensure a safe level of exposure we require:

$$\sum_{\lambda \in SM} \left(\frac{Qf_\lambda}{AEL_\lambda} \right) \cdot \tau_\lambda \leq 1. \quad (2)$$

Equation (2) will form one of the constraints used in the mathematical program used to optimally design LEP. The goal is to provide various criteria by which to optimize the LEP, while providing safe exposure to the multi wavelength laser. One option is to maximize visibility through the eyewear, and for that we require a scalar performance measure that captures *quality of vision* as a function of transmittance at each wavelength. The *spectral luminous efficiency* assigns a weight to each wavelength in the spectrum that captures its relative contribution to visibility in a typical environment for a user⁶.

There are, theoretically, infinite combinations of OD for each multi wavelength laser that would provide a solution to Equation (2). There may be some modes that are already considered safe individually, i.e.: $(Qf_\lambda / AEL_\lambda) \leq 1$. For the other modes, the OD applied for that mode has a minimum such that $10^{-D_\lambda} \cdot (Qf_\lambda / AEL_\lambda) = 1$. Beyond this, there are no constraints on how we apply optical densities.

From a more practical standpoint, there may be other considerations such as the cost involved in producing OD values for specific wavelengths or the desire to maximize transmittance over some wavelength band. In this case, we may correspondingly assign a positive weighting value to each mode V_λ , $\lambda \in \Lambda$. If we are not concerned with discriminating between the wavelengths being emitted, a uniform weight of 1.0 may be applied for each V_λ , which would in turn simply minimize the sum of the optical densities for each mode.

EVAULATION PARAMETERS

Several parameters must first be gathered in order to determine the optimal OD requirement for each mode of a multi wavelength laser exposure.

Qf_{λ} – The total amount of energy expected to enter the applicable limiting aperture for each mode. This value takes all modifiers into account besides OD, including but not limited to atmospheric attenuation, transmittance and magnification inherent to aided viewing scenarios, beam spread due to laser geometry, exposure duration, etc. The ANSI Z136.1-2000¹ provides guidelines and calculations for finding this value given a particular or a worst-case scenario.

AEL_{λ} – The accessible emission limit, or desired energy exposure considered to make this particular mode of the laser safe. The ANSI Z136.1-2000¹ may be used to find the MPE and use that to determine the worst-case AEL for the scenario. A less conservative value may also be used here depending on the user's needs.

The values for AEL_{λ} and Qf_{λ} may have been determined in Watts, Joules, or some other related unit of power or energy. The units used are not important, so long as the units used are consistent between AEL_{λ} and Qf_{λ} such that the ratio between them, $(Qf_{\lambda}/AEL_{\lambda})$, is meaningful.

V_{λ} – A relative weight for each wavelength that will help to discriminate the assignment of OD values. This value must be greater than zero at each wavelength, and must be constant with respect to the OD assigned. Modes with a relatively higher V_{λ} will be assigned a lower OD such as to maintain the objective function, Equation (3). Regardless of this weight, the final solution will always provide a safe exposure limit if the wavelength were considered independently, i.e. $10^{-D_{\lambda}} \cdot (Qf_{\lambda}/AEL_{\lambda}) \leq 1$. If we are not concerned with discriminating between the modes, a weight of 1.0 may be applied for each V_{λ} , which would simply provide a solution that minimizes the sum of the optical densities for each wavelength.

An example weighting value V_{λ} , which represents the *spectral luminous efficiency* curve shown below in Figure 1 as a function of wavelength,^{6,7} can be used to maximize visibility. Because the human eye senses different colors (wavelengths) with varying acuity, when the wavelengths are outside the visible spectrum, visual acuity is effectively zero, but peaks around 550 nm. In our research, we have the option of weighting the transmittance, τ_{λ} , at each wavelength by V_{λ} and therefore maximize visibility. We note that if these values are used, it is important to provide some minimal value for wavelengths on the “wings” of the curve that is greater than zero. The examples used in the paper will use values from V_{λ} shown in Figure 1.

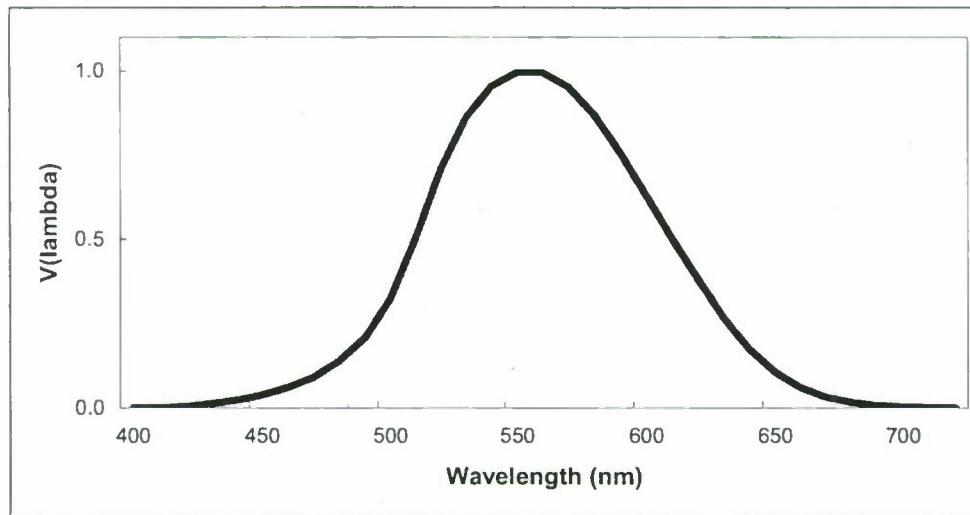


Figure 1. Visible Luminous Efficiency for the Human Eye

PROCEDURE FOR OPTICAL DENSITY DETERMINATION

Optimal design of LEP for multi wavelength lasers begins with the same calculations and requirements as outlined in the ANSI Z136.1-2000¹ for single wavelength exposures. However, as previously indicated, when laser exposures involve more than one wavelength, the calculations are no longer straightforward, and so, in this paper mathematical programming and algorithmic approaches are used to determine optimal LEP design.

Mathematical Programming Approach

Mathematical programming is a set of models, methods and theory that deals with optimization of typically complex systems. The field includes linear programming, nonlinear programming and integer programming. It primarily concerns minimizing or maximizing an objective function subject to one or more constraints. Generally, mathematical programs seek to optimize some objective through the selection of a set of values for the decision variables⁸. These variables are constrained by various conditions and restrictions. Mathematical programming chooses levels of the decision variables so that the objective is optimized while maintaining the integrity of the constraints. The problem of optimal LEP design is first formulated as a nonlinear optimization algorithm and solved using GAMS⁴. An objective function of this specific problem involves finding a solution which minimizes OD for each wavelength and maximizes visibility, while maintaining the constraint ensuring a safe exposure scenario. Although the logarithmic (not linear) relationship between OD and transmittance appears to cause significant difficulties, the specifics of this problem alleviate this complexity.

The problem was first formulated as a mathematical programming model. Because the constraint that ensures a safe viewing condition, Inequality (4), is nonlinear in form, the

formulation is considered a nonlinear program. The decision variable OD_λ is optimized according to the following model formulation:

$$\text{minimize} \quad Z = \sum_{\lambda \in \Lambda} D_\lambda \cdot V_\lambda \quad (3)$$

$$\text{subject to} \quad \sum_{\lambda \in SM} \left(\frac{Qf_\lambda}{AEL_\lambda} \right) \cdot 10^{-D_\lambda} \leq 1 \quad (4)$$

$$\text{and} \quad D_\lambda \geq 0, \lambda \in \Lambda. \quad (5)$$

The objective function, Equation (3), seeks to optimize the OD according the user-specified weighting values for V_λ . When V_λ is the spectral luminous efficiency, the visibility of the LEP is maximized because the spectral luminous efficiency data, V_λ , positively weights transmittance $\tau_\lambda = 10^{-D_\lambda}$ over the visible wavelengths. In this situation, the objective function prefers lower optical densities in the visible wavelengths and therefore more visible light will be transmitted through the LEP to the user's eye.

Inequality (4) is the constraint that ensures the overall exposure is safe. As described previously, a multi wavelength laser exposure is safe when Inequality (4) holds for the wavelengths being emitted ($SM \subset \Lambda$). When multiplied by the transmittance $\tau_\lambda = 10^{-D_\lambda}$, the ratio of total energy delivered to the accessible emission limit must be less than one. It is through this constraint that we have flexibility in setting the optical densities. For example, when maximizing visibility we can compensate for lower optical densities in the visible spectrum with higher optical densities in the non-visible, or out-of-band wavelengths.

GAMS⁴ was configured using the formulation above and run against a number of representative laser exposures. The results of the analysis and their validation are described in Section 5.

Algorithmic Approach

In an effort to utilize the mathematical analysis formulated and run in GAMS⁴, an algorithm was independently formulated for the specific problem of hazard analysis. This algorithm is sufficiently simple such that an analysis can be conducted with a handheld calculator, or with basic programming skills can be implemented for use in software applications. The algorithm is limited in scope when compared to the capabilities of GAMS, but should suffice for most laser safety analysis requirements. It is instructive to examine the development of our algorithm through the use of an illustrative example. Working through the exercise provides insight to the logic applied to derive the final equations of the method.

After assembling all of the output parameters of the laser in question, the initial step in the algorithm is to determine if the laser is already safe without any additional OD from LEP. This involves simply summing up the ratios of each (Qf_λ/AEL_λ) and checking to see if the sum is less than or equal to one, i.e. $\sum_\lambda (Qf_\lambda/AEL_\lambda) \leq 1$. The remainder of the procedure assumes that this is not the case. The goal of this algorithm then is to fulfill (3) and (4), restated as:

$$A = \sum_\lambda 10^{-D_\lambda} \cdot \frac{Qf_\lambda}{AEL_\lambda} = 1, \text{ while minimizing } Z = \sum_\lambda D_\lambda \cdot V_\lambda.$$

If we consider these equations for each mode (wavelength) independently and take the first derivative of each with respect to D_λ , we can easily see the effect of increasing the OD for each mode:

$$\begin{aligned} A_\lambda &= 10^{-D_\lambda} \cdot \frac{Qf_\lambda}{AEL_\lambda} \\ \dot{A}_\lambda &= 10^{-D_\lambda} \cdot \frac{Qf_\lambda}{AEL_\lambda} \cdot -\ln(10) \end{aligned} \quad (6)$$

$$\begin{aligned} Z_\lambda &= D_\lambda \cdot V_\lambda \\ \dot{Z}_\lambda &= V_\lambda. \end{aligned} \quad (7)$$

Therefore, our approach is to set the goal of the algorithm to increase OD on the mode that has the greatest impact in decreasing A to 1 while having the least impact in raising Z . To help us do that, we will introduce a weighting function that combines (6) and (7):

$$W_\lambda(D_\lambda) = \frac{\dot{A}_\lambda}{\dot{Z}_\lambda} = 10^{-D_\lambda} \cdot \frac{Qf_\lambda}{AEL_\lambda \cdot V_\lambda} \cdot -\ln(10). \quad (8)$$

Since we are only using the function for comparison between modes, we can factor out the common constant elements to get:

$$W_\lambda(D_\lambda) = 10^{-D_\lambda} \cdot \frac{Qf_\lambda}{AEL_\lambda \cdot V_\lambda}. \quad (9)$$

The most beneficial effect of adding small increments of OD to individual modes of the laser is adding OD to the mode with the greatest W value for its current D_λ . To help illustrate this, we will use the following example of a multi-mode laser (this is the laser named “C” in the results section). We have also used the spectral luminous efficiency curve described above to obtain illustrative values for V . Table 1 illustrates the initial values for the algorithm using laser “C”.

Table 1. Initial Algorithm Values for Example Laser “C”

Wavelength (nm)	Qf (W)	AEL (W)	V
458	0.02	6.50E-04	5.56E-02
488	0.1	6.50E-04	1.94E-01
514	0.2	6.50E-04	5.86E-01
640	0.05	6.50E-04	1.75E-01

Examination of Table 1 quickly shows that each of the modes of this laser is hazardous ($Qf/AEL > 1$ for each mode), so each mode has a minimal OD value (D_{min}) greater than zero. The values of D_{min} for each mode are shown in Table 2.

Table 2. Individual Mode Hazard Analysis Values of Dmin and Weighting Factors for Example Laser “C”

Wavelength (nm)	Qf (W)	AEL (W)	V	T	D _{min}	W
458	0.02	6.50E-04	5.56E-02	3.25E-02	1.49	18.0
488	0.10	6.50E-04	1.94E-01	6.50E-03	2.19	5.15
514	0.20	6.50E-04	5.86E-01	3.25E-03	2.49	1.71
640	0.05	6.50E-04	1.75E-01	1.30E-02	1.89	5.71

Applying a minimal OD of D_{λ} from Table 2 makes each mode safe (A_{λ} for each mode is 1), but the laser as a whole is still not safe because there are four modes. We see that Equation (6) would yield a value of 4 in this case. We still need to increase the OD for one or more modes. We can choose the mode to work on by computing $W(D)$.

If we solve for $W(D_{min})$ for each mode, we can find the mode with the greatest W that will allow us to increase its OD with the greatest impact on A and the least impact on Z . Table 2 illustrates the computation of W for each mode based upon the initial hazard analysis.

From the values in Table 2, we see that the mode to address is the one with the highest W value based on D_{min} . Raising the OD for the 458-nm mode a small amount will have the greatest impact on A and the least impact on T . But how much can we raise this OD? Looking at Equation (9), we will note that W is monotonically decreasing as D increases for all values that are greater than or equal to 0. This means continually raising the OD for one mode will have a decreasingly beneficial effect. We can increase the OD of the 458-nm mode until $W(D)$ is equal to the next lowest W value; in this case, the 640-nm mode has a W value of 5.71.

Using Equation (9) and solving for D_λ we obtain Equation (10).

$$D_\lambda = -\log\left(\frac{W_\lambda \cdot AEL_\lambda \cdot V_\lambda}{Qf_\lambda}\right) \quad (10)$$

In this case, we get a new value of D_λ for the 458-nm mode of 1.99. We now compute new values of A and Z to determine if the new combination of OD values will make the entire laser safe. Table 3 represents an updated configuration of parameters, along with the values of A and Z .

Table 3. Individual Mode Hazard Analysis Values of Dmin and Weighting Factors for Example Laser “C” after One Algorithm Iteration

<i>Wavelength (nm)</i>	<i>Qf (W)</i>	<i>AEL (W)</i>	<i>V</i>	<i>D_λ</i>	<i>W</i>
458	0.02	6.50E-04	5.56E-02	1.99	5.71
488	0.1	6.50E-04	1.94E-01	2.19	5.15
514	0.2	6.50E-04	5.86E-01	2.49	1.71
640	0.05	6.50E-04	1.75E-01	1.89	5.71

$$A = 3.3177, Z = 2.3288$$

Unfortunately, this still does not yet provide for a safe exposure ($A > 1$), so we need to continue. We can now look to the next highest W value, and increase the OD at the 458-nm *and* the 640-nm modes to the point where their W values are the same. In this case, the 488-nm mode has a W value of 5.15. We can apply (10) to the 458-nm *and* 540-nm modes to find new D_λ values for both of them. The results are shown below in Table 4. We see that the value of A is indeed reduced, indicating that the exposure is less hazardous. However, A remains greater than 1, indicating that a hazard does indeed remain.

Table 4. Individual Mode Hazard Analysis Values of Dmin and Weighting Factors for Example Laser “C” after Two Algorithm Iterations

<i>Wavelength (nm)</i>	<i>Qf (W)</i>	<i>AEL (W)</i>	<i>V</i>	<i>D_λ</i>	<i>W</i>
458	0.02	6.50E-04	5.56E-02	2.03	5.15
488	0.1	6.50E-04	1.94E-01	2.19	5.15
514	0.2	6.50E-04	5.86E-01	2.49	1.71
640	0.05	6.50E-04	1.75E-01	1.93	5.15

$$A = 3.1874, Z = 2.3332$$

At every stage of iteration, we are using W to find the optimal mode and only increasing the OD for that mode which has the greatest impact in decreasing A with the least impact on increasing Z . We could continue with this method until we reach $A = 1$.

We see that if we had to increase the OD for every mode, then every mode would eventually share the same W value. This indicates that we can solve for this W value (W') from the very beginning and use Equation (10) to find the OD that would provide that W'

value. Using $A = \sum_{\lambda} 10^{-D_{\lambda}} \cdot \frac{Qf_{\lambda}}{AEL_{\lambda}} = 1$ and replacing OD using (10) we get:

$$\sum_{\lambda} 10^{-\left(\log\left(\frac{W' \cdot AEL_{\lambda} V_{\lambda}}{Qf_{\lambda}}\right)\right)} \cdot \frac{Qf_{\lambda}}{AEL_{\lambda}} = 1. \quad (11)$$

This equation simplifies to:

$$\sum_{\lambda} W' \cdot V_{\lambda} = 1, \text{ or } W' \cdot \sum_{\lambda} V_{\lambda} = 1 \quad (12)$$

for cases of constant values of W' . This indicates that we can again simplify to:

$$W' = \frac{1}{\sum_{\lambda} V_{\lambda}}. \quad (13)$$

So for our example laser “C”, we can find $W' = 0.99$ from our values of V_{λ} . This gives the resulting optical densities shown below in Table 5, and a safe multi wavelength laser exposure.

Table 5. Individual Mode Hazard Analysis Values and Weighting Factors for Example Laser “C” Applying Equation (13) in the Development of our Algorithm

Wavelength (nm)	Qf (W)	AEL (W)	V	D _λ	W
458	0.02	6.50E-04	5.56E-02	2.75	0.99
488	0.1	6.50E-04	1.94E-01	2.90	0.99
514	0.2	6.50E-04	5.86E-01	2.72	0.99
640	0.05	6.50E-04	1.75E-01	2.65	0.99

$$A = 1.0, Z = 2.7762$$

For a multi-mode laser where every mode is hazardous, the solution then is to find W' using Equation (13), then find the value of D_λ for each mode using Equation (10). If there are one or more modes that are *not* hazardous, we may not know if those modes require any OD as part of the safety analysis. As an example of this condition, we take the example laser “C” and change the value of Qf for the 640-nm mode to be non-hazardous and evaluate the optimal OD requirements. We begin with the parameters in Table 6. We can assume an initial OD of zero for each mode and use Equation (9) to solve for the W values for each mode. This analysis is also shown in Table 6.

Table 6. Initial Parameters for a Modified Laser “C” Assessment of Optical Density Requirements

Wavelength (nm)	Qf (W)	AEL (W)	V	D_{min}	W
458	0.02	6.50E-04	5.56E-02	0	553.4
488	0.1	6.50E-04	1.94E-01	0	793.0
514	0.2	6.50E-04	5.86E-01	0	525.1
640	6.0E-04	6.50E-04	1.75E-01	0	5.3

In this situation, we will note that the $W(0)$ values for the mode are still greater than the W' value calculated using Equation (13), which was 0.99. This means that even though the 640-nm mode is not hazardous, the optimal solution would still involve raising the OD of this mode some amount. Once again, we apply Equation (10) to calculate the OD for each mode in order to provide for a safe exposure. We obtained the results summarized by Table 7.

Table 7: Algorithm Results for Assessment of a Modified Laser “C”

Wavelength (nm)	Qf (W)	AEL (W)	V	D_λ	W
458	0.02	6.50E-04	5.56E-02	2.75	0.99
488	0.1	6.50E-04	1.94E-01	2.90	0.99
514	0.2	6.50E-04	5.86E-01	2.72	0.99
640	6.00E-04	6.50E-04	1.75E-01	0.73	0.99

$$A = 1.0, Z = 2.4397$$

If one or more of the $W(0)$ values for the mode are less than or equal to the W' value, then these modes do not need any OD to reach an optimal solution. When this occurs, it is appropriate to remove those modes from the equation and solve for a new W' value. This can be done until all of the remaining modes have a $W(0)$ value of greater than W' . As a final case study, we modify our example laser “C” to present the situation when no OD is required at one wavelength. Our initial parameters are presented in Table 8.

Table 8. Initial Parameters and for a Second Modified Laser “C” Assessment
of Optical Density Requirements

Wavelength (nm)	Qf (W)	AEL (W)	V	D_{min}	W
458	0.02	6.50E-04	5.56E-02	2.71	1.08627
488	0.1	6.50E-04	1.94E-01	2.86	1.08627
514	0.2	6.50E-04	5.86E-01	2.68	1.08627
640	6.00E-05	6.50E-04	1.75E-01	0	N/A

$$A = 1.0, Z = 2.2760$$

In this example, the Qf value for the 640-nm mode is lowered such that its W value is less than the 0.99 that was calculated for W' . This indicates that the optimal solution would not include any OD for this mode. We also cannot have a negative OD value, so we should effectively remove this mode from the computation.

First, we calculate the effect that this mode would have on A . In this case,

$$A_\lambda = 10^{-D_\lambda} \cdot Qf_\lambda / AEL_\lambda \text{ for the 640-nm mode, such that } A_\lambda = \frac{6.00E-5}{6.50E-4} = 0.09231. \text{ We}$$

effectively want the sum of A_λ of the other three modes to make the difference between 1.0 and 0.0923, or 0.9077. Using this new value for our desired A' , we will find the new

optimal W' using a derivation of Equation (13): $W' = 0.90769 / \sum_{\lambda \in \Lambda} V_\lambda$ for all of the

remaining modes. So in this case we have: $W' = 1.08627$. We then use Equation (10) to find the OD for each of the remaining modes, giving us the optimal solution shown in Table 9.

To re-cap, the procedure can be summarized as follows:

- (1) Find W_λ for each mode of the laser, starting with an OD of 0 for each mode. Using the simplified Equation (9).

$$W_\lambda(0) = \frac{Qf_\lambda}{AEL_\lambda \cdot V_\lambda}. \quad (14)$$

- (2) Find the optimal W' for the laser using (13):

$$W' = \frac{1}{\sum_{\lambda} V_\lambda}. \quad (15)$$

Compare W' to the W_λ for each mode. If there are any $W_\lambda < W'$, these modes will not have any increase in D_λ . Call this subset of Λ, J and remove them from the calculation of W using:

$$W' = \frac{1 - \sum_j \frac{Qf_j}{AEL_j}}{\sum_\lambda V_\lambda - \sum_j V_j}. \quad (16)$$

If this new W' is still greater than W_λ of one of the remaining modes, remove that mode in the same way as the step above until there is some sub-set of the original modes where $W_\lambda \geq W'$ for each mode.

(3) For every mode remaining, find the required optical density using:

$$D_\lambda = -\log\left(\frac{W' \cdot AEL_\lambda \cdot V_\lambda}{Qf_\lambda}\right). \quad (17)$$

We note that in case of equally-weighted $V_\lambda = 1.0$, then the trivial solution of increasing each optical density by $\log_{10}(N)$, where N is the number of individual laser modes.

DISCUSSION

The two methods were checked for agreement and accuracy using data from five example lasers; the results are shown in Table 9. The first column of the table below contains the wavelengths that each example laser emits. The second and third columns are the Qf_{λ} and AEL_{λ} as calculated according to the ANSI Z136.1-2000¹. The resulting optimal optical densities at each wavelength are shown for the two methods: nonlinear programming and the algorithmic approach in the fourth and fifth columns. The safety check is a manual calculation of the value of the constraint required for a safe exposure from Inequality (2). Recall that the value of the safety constraint, Inequality (2), must be less than or equal to 1 in order to ensure a safe exposure.

Table 9. Summary of Data Comparing Mathematical Programming Results and the Algorithm for Five Sample Lasers

Laser Input Paramters			Optical Density Results	
			<i>Nonlinear programming</i>	<i>Algorithmic approach</i>
“A”	Qf(J)	AEL(J)		
532 nm	1.00E-03	5.00E-07	3.301	3.302
1064 nm	5.00E-04	5.00E-06	10.000	4.945
			<i>safety check</i>	<i>1.000</i>
“B”	Qf(J)	AEL(J)		
532 nm	1.00E-01	6.50E-04	2.187	2.188
860 nm	5.00E-01	1.40E-03	10.000	5.498
			<i>safety check</i>	<i>1.000</i>
“C”	Qf(J)	AEL(J)		
458 nm	2.00E-02	6.50E-04	2.748	2.748
488 nm	1.00E-01	6.50E-04	2.903	2.903
514 nm	2.00E-01	6.50E-04	2.725	2.725
640 nm	5.00E-02	6.50E-04	2.648	2.648
			<i>safety check</i>	<i>1.000</i>
“D”	Qf(J)	AEL(J)		
430 nm	1.00E-02	1.00E-05	4.829	4.829
530 nm	5.00E-04	6.50E-04	0.000	0.000
640 nm	5.00E-02	6.50E-04	2.552	2.552
			<i>safety check</i>	<i>1.000</i>
“E”	Qf(J)	AEL(J)		
800 nm	1.00E-02	1.60E-02	2.085	0.097
1064 nm	1.00E-03	5.00E-06	2.303	2.602
			<i>safety check</i>	<i>1.000</i>

The first results column presented in Table 9 are based on solving the nonlinear program using the MINOS NLP solver, which we call from the GAMS mathematical programming language^{9,4}. These are compared to the results from the algorithmic approach implemented in the C++ programming language^{5,10}. We see by the overly safe OD of 10 that resulted in a few of the examples (with the nonlinear mathematical program) that this particular formulation is not the best for that method, but provides a basis and comparison for the algorithmic approach.

By comparing the optical densities determined to be optimal for each example laser, it is clear that the wavelengths in the visible spectrum are being favored according to the weights set by the spectral luminous efficiency values. These weights prefer lower optical densities for the visible wavelengths and the models compensate by putting higher OD on the non-visible wavelengths. This satisfies the goal of maximizing visibility while providing a safe viewing condition for multi wavelength exposures.

CONCLUSIONS

This paper presents a brief tutorial regarding the computation of optimal optical density values for multi wavelength lasers. It is our hope that with this information, hazard analyses may be conducted with more understanding and greater confidence in the results. The purpose of this work was to transition the optimization methods used in nonlinear programming into a versatile format that can be integrated into existing laser hazard software, LTMC.

The Laser Threat Modeling Components (LTMC) is an ANSI C++ library developed by AFRL/HEDO to encapsulate the computational logic used by LHAZ and LRMS to conduct basic ANSI Z136.1-2000 analyses. It also contains additional logic for computing ED50's, hazard distances, recommended optical densities, and irradiance/radiant exposure at range. It has been compiled under Windows, Mac OS X, and Linux. The current implementation supports single wavelength emitting lasers. For these, as indicated earlier, a recommended optical density is computed to ensure exposure at a specified minimal range does not exceed the ANSI Z136.1-2000 maximum permissible exposure (MPE) threshold. Plans for the next generation LHAZ application include support for multi wavelength laser hazard analyses. In conjunction with this plan, an implementation of the OD optimization algorithms discussed here will be completed as an extension to LTMC's support. Additionally, a graphically user interface within LHAZ will be supplied to access this LTMC capability.

APPENDIX

This appendix contains the C++ source code for the algorithmic approach detailed above.

```
#include "SMinOD.h"
#include <vector>
#include <math.h>
using namespace std;

/// Looks through a vector of integers to see if index exists
bool Contains(const vector<int>& UsedI, int index)
{
    for (unsigned i = 0; i < UsedI.size(); ++i)
    {
        if (UsedI[i] == index) return true;
    }
    return false;
}

/// The function that does the work. A vector of SMinOD instances is passed
in.
/// The OD member for each instance is populated based on the algorithm
described in the documentation
void SMinOD::MinimizeOD(vector<SMinOD>& V)
{
    // Watch for pedantic case of already being under, this also checks for
0 elements
    double A1 = 1.0;
    double safety = 0.0;
    int c = (int)V.size(); int i = 0;
    for (i = 0; i < c; ++i)
    {
        V[0].OD = 0;           // Populate the required OD at 0
    for the pedantic case
        safety += V[i].Qf / V[i].AEL;
    }
    if (safety <= 1.0) return;

    // Look for a single mode, find the simple OD for that mode
    if (c == 1)
    {
        V[0].OD = -1.0 * log10(V[0].AEL/V[0].Qf);
        return;
    }

    // Watch for modes that have maximum OD, see if it will drop the value
below 1
    for (i = 0; i < c; ++i)
    (
        if (V[i].VisC > 1.0)
        {
            V[i].OD = V[i].VisC;
            safety -= V[i].Qf / V[i].AEL;
        }
    }
}
```

```

        safety += (V[i].Qf * pow{10.0, -V[i].OD}) / V[i].AEL;
    }
}

if (safety > 1.0)
{
    // Safety is still high, max the OD on what we have and use
W for the other modes
    for (i = 0; i < c; ++i)
    {
        if (V[i].OD == 0.0)
            V[i].OD = -V[i].Qf / {V[i].AEL * V[i].VisC};
        else
            A1 -= (V[i].Qf * pow{10.0, -V[i].OD}) /
V[i].AEL;
    }
}
else if (safety < 1.0)
{
    // Safety is TOO low. Make all of the other numbers 0 and
just solve for these maxOD's
    if (V[i].OD == 0.0)
        A1 -= V[i].Qf / V[i].AEL;
    else // Weight based on the max OD
    {
        V[i].VisC = 1.0 / V[i].OD;
        V[i].OD = -V[i].Qf / (V[i].AEL * V[i].VisC);
    }
}
else return;      // Probably won't happen, but just in case max
OD makes us perfectly safe...

// Loop until we find a W that works for the modes, and use 0 for the
OD of the others
double W;
double minW = 1e6;
do
{
    minW = 1e6;
    W = 0.0;
    int mini = -1;
    for (i = 0; i < c; ++i)
    {
        if (V[i].OD < 0.0)
        {
            W += V[i].VisC;
            if (-V[i].OD < minW)
            {
                mini = i;
                minW = -V[i].OD;
            }
        }
    }
    W = A1 / W;
    if (W > minW)
    {
        V[mini].OD = 0.0;
        A1 -= V[mini].Qf / V[mini].AEL;
    }
}

```

```
)  
while (W > minW);  
  
// Find the OD for the other modes  
for (i = 0; i < c; ++i)  
{  
    if (V[i].OD < 0.0)  
        V[i].OD = -1.0 * log10((W * V[i].AEL * V[i].VisC) /  
V[i].Qf);  
}  
}
```

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